Using Dynamic Geometry Software to Encourage 3D Visualisation and Modeling

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Abstract

This study explores the integration of 3D modeling and printing in teacher education as a means to foster mathematical modeling and spatial visualization skills. The objective was to investigate how graduate students engage in iterative modeling cycles using the online CAD tool Tinkercad and 3D printers within a project-based learning environment. A total of 13 master's students participated in a five-week course in which they individually designed 3D models representing typical dishes from their regions, combining mathematical reasoning with technological proficiency. Data collection involved student dossiers, screenshots, physical models, and reflective journals, analyzed qualitatively to trace modeling phases and strategies. Results highlight the effectiveness of iterative modeling in developing spatial and mathematical skills, with students performing an average of 2.4 modeling cycles. Two detailed case studies illustrate different modeling pathways: one based on a physical object with direct measurement, and another based on a visual reference requiring abstraction. The findings underscore the pedagogical value of combining structured and open-ended design tasks to promote both precision and creativity. Visualization, digital tools, and embodied modeling actions emerged as key elements supporting engagement and conceptual understanding. The study concludes that exposing future educators to both fixed-referent and open-ended modeling experiences enriches their understanding of mathematical modeling and better prepares them for diverse teaching contexts.

1. Introduction

Mathematical modeling is a process that allows us to connect mathematics with the world around us, applying and inventing mathematics to solve problems [6]. It is configured as a mathematical competence developed through the creation of mathematical models explaining real-life phenomena [14].

The inclusion of digital tools produces new opportunities for the learning and teaching of mathematical modeling, as they allow multiple and interactive representations of abstract concepts, providing new ways of visualizing, understanding, evaluating, and interpreting real-world situations [10]. Research on the educational potential of digital technologies in mathematical modeling is attracting recent interest, providing evidence of their usefulness in the modeling process, and revealing that the benefits outweigh the difficulties in successfully using the technology in modeling [5].

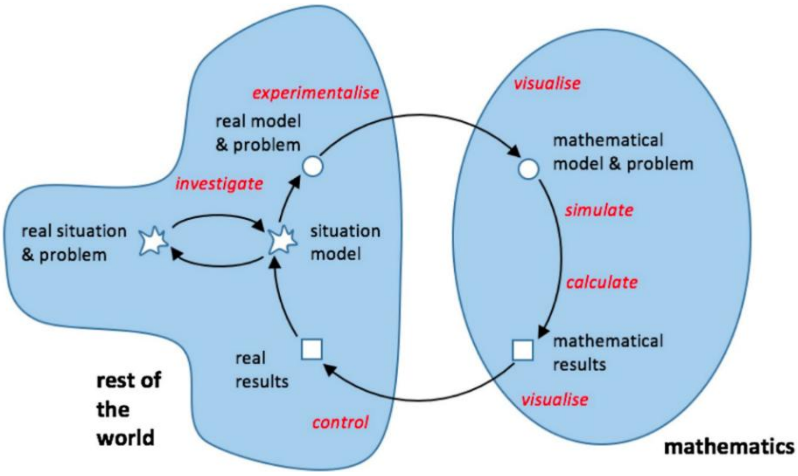

Figure 1. Modeling cycle of Greefrath [9], based on Blum and Leiß [4].

The interrelation between the mathematical and the real world can be described from the model of Blum and Leiß [4] by means of the so-called modeling cycle (see Figure 1). The model consists of 7 steps: it starts with a real situation or problem that, to be solved, requires an understanding and simplification of the situation to establish a *real model* of the problem. This model is translated into the mathematical world through *mathematization*, and the mathematical work (calculations, reasoning, equation solving, etc.) produces *mathematical results,* which can be interpreted and validated as *real results* in the real world. This process is cyclic, since the validation of these results will determine whether the desired result has been reached, either concluding with the presentation of these results, or with the reformulation of the model when another round must begin [2,3].

The employment of digital tools during a modeling process influences each part of the cycle [9]. Some examples are shown decorating in red the modeling cycle in Figure 1, including the steps *investigate*, *experimentalize*, *simulate*, *calculate*, *visualize,* and *control,* as well as the steps of *construct*, *draw,* and *measure* from Figure 2 [10, p. 236] in the context of a Dynamic Geometry Software.

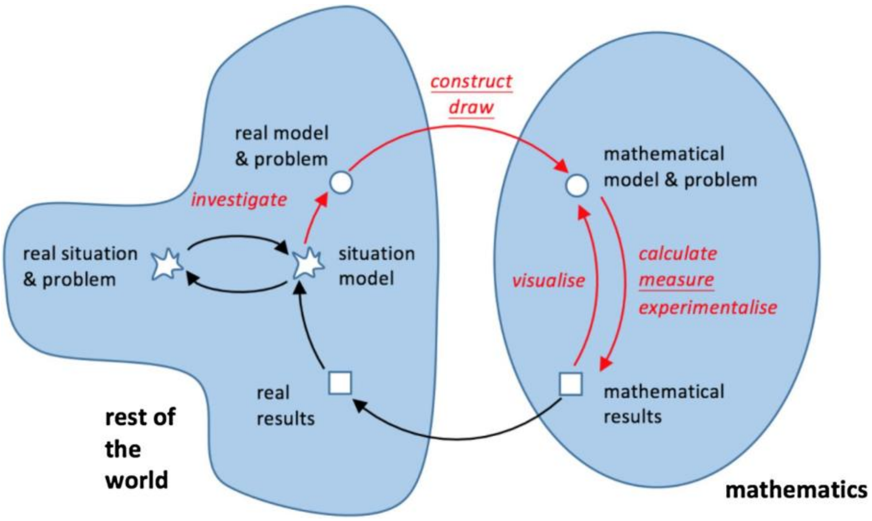

Figure 2. Modeling cycle of Greefrath [10], in the context of a Dynamic Geometry Software.

An example of an activity that develops mathematical modeling skills is the so-called 3D modeling and printing activities [12]. These activities offer students a real problem that they have to solve through the creation of prototypes or three-dimensional models with 3D designing tools that can be replicated and validated thanks to the use of 3D printing. When a fixed object is provided, students engage in structured modeling that emphasizes accuracy and adherence to given constraints. In contrast, open-ended design challenges encourage exploration and creativity, pushing students to develop spatial and mathematical competencies in more flexible and innovative ways. This distinction aligns with the continuum of embodied spatial-mathematical learning activities proposed in [17]. This type of project is considered a tools with great educational potential that improve both participation and motivation in the learning process, especially in the context of STEAM education [1,12].

From a mathematical point of view, these are activities that encourage students' visualization and spatial reasoning using computer-aided design CAD software [7,13]. They allow students to work collaboratively, combine theory with practice, and provide teachers with a tool to design manipulative materials [8]. In addition, the process of validating the models with real objects allows students to self-assess their own results [1]. Furthermore, in open-ended 3D prototyping activities, students are encouraged to define the problem themselves, generate and evaluate possible design alternatives, and iteratively improve their models, which fosters higher-order thinking and creativity [18].

Finally, we would like to emphasize the need to promote teacher training programs that include STEAM activities and projects that improve knowledge about this approach and provide the necessary tools to implement them in the classroom [16]. In particular, and related to the present work, several authors argue that both modeling and 3D printing activities represent enriching activities for future mathematics teachers [11].

The primary objective of this study was to investigate the development of mathematical modeling and visualization skills in graduate students through a project-based learning approach using the 3D modeling software Tinkercad and printing technology.

2. Methodology

2.1 Participants

This study involved a group of 13 graduate students enrolled in the subject *The use of space in Mathematics Education* of the Master of Innovation in Specific Didactics at Universidad Autónoma de Madrid (UAM) during 5 weeks in the academic year 2023/24. Among the participants, 11 of them graduated in Early Childhood and/or Primary Education, and the other 2 had no background in

Education. The group consisted of 9 females and 4 males, with ages ranging from 22 to 35 years. None of the participants had previous experience with 3D printing or the software Tinkercad.

2.2 Materials

Two main resources were used to carry out the project: the three-dimensional modeling program Tinkercad and 3D printers.

Tinkercad (https://www.tinkercad.com/), an online 3D CAD design tool, was used by students to create and simulate their 3D models. This software is specifically tailored for educational purposes, offering a user-friendly interface to learn and apply the 3D principles of design quickly. Its accessibility via web browsers eliminates the need for high-performance computing equipment, making it ideal for educational settings. The software supports direct printing capabilities, allowing designs to be easily exported and prepared for 3D printing. Tinkercad includes a variety of built-in tools that encourage spatial reasoning and support the development of geometric thinking. These include:

- *Insertion of solids*: Users can drag and drop geometric primitives (like cubes, spheres, or cylinders) onto the workspace to begin shaping their models.
- *Rescaling and dimensioning*: Shapes can be resized manually or by entering precise values, allowing control over proportions and measurements.
- *Combining elements*: Different solids can be merged to form more complex objects or subtracted from each other to hollow or shape parts.
- *Alignment tools*: Objects can be automatically aligned along horizontal, vertical, or depth axes to maintain symmetry and coherence.
- *Cutting and carving*: By assigning the "hole" property to a shape, users can subtract it from others.
- *Rotation features*: Shapes can be rotated around the three spatial axes, helping in adjusting orientation and fitting structures together.
- *Measurement utilities*: A ruler can be placed to assess distances and positions within the 3D model.
- *Multi-angle viewing*: The camera can be moved freely around the model, which allows users to inspect their work from different perspectives.

A detailed overview of these design possibilities can be found in [17].

Two 3D printers (a Creality Ender-5 and a Prusa i3 Mk3/Mk3s) were made available in the university's technology lab, providing students the opportunity to bring their digital models into tangible forms. These printers were capable of printing designs with PLA plastics, which are commonly used in educational environments due to their cost-effectiveness and ease of use.

In addition to Tinkercad and 3D printers, students who based their modeling on visual references (such as photographs or sketches) used auxiliary tools to support their design process. Some of them used physical rulers to take measurements from real objects, or searched for standard dimensions online to approximate the proportions of the object. When modeling organic objects—like the grapevine leaf described in section 3.2—students sometimes created intermediate sketches on paper to analyze forms before reconstructing them geometrically. These strategies enhanced the accuracy of modeling when direct measurement was not possible and reinforced spatial reasoning and abstraction skills.

Students compiled dossiers that included detailed accounts of their design processes, mathematical calculations, and iterations of the 3D models. These dossiers served as a concrete record of the application of mathematical concepts and the evolution of their project work, from initial design to final validation. The project dossiers also included reflective journals; in them, students talked about some of the difficulties, challenges and lessons learned.

2.3 Procedure

The students were asked to carry out a modeling project of their choice, where they had to jointly choose a real space/object to replicate in 3D. All the selected spaces/objects must have a clear relationship among them, as well as keeping the proportion. The project was called *Foods of the World*, where each student had to recreate a typical dish from their region or country in order to reflect and share the different customs and cultures represented in the group. Throughout the course, 4 of the sessions were used to resolve doubts, to correct frequent modeling errors and to reorganize the size and scale of the figures so that the overall project would have coherence. All the objects were shown in a final exhibition.

The main objective was for the students to learn how to use the Tinkercad tool and to perform modeling cycles (Figure 2) throughout the design process: choosing the object and obtaining its real measurements, designing the 3D model, 3D printing, validating the model, re-designing the 3D model, re-printing the 3D model, re-validation, etc., based on the Greefrath model [10]. The process was reflected in a dossier that included the steps followed, and the mathematical elements contained in their 3D designs.

Throughout the sessions the students had constant experimentation and knowledge of both the Tinkercad software and the 3D printing process (3D design, lamination and printing phase). Most of the time the students worked autonomously on their models and when they finished a model it was printed and proceeded to validation. A modeling cycle is considered to have been completed when the student obtains the 3D printed object and can therefore carry out its validation with the real starting object and with the rest of the objects in the project.

2.4 Data Collection and Analysis

Participants documented the various phases of the modeling process in their project dossiers. These dossiers included screenshots from the Tinkercad software, capturing the different stages, refinements and re-designs of their 3D models. Additionally, participants included photos of the real-world objects they aimed to model, alongside images of the 3D printed versions of their designs. The dossier projects detailed the types of geometric figures intended for modeling and provided insight into the cognitive and affective experiences of the participants. This comprehensive visual documentation was accompanied by reflective journals, where participants recorded their thoughts and experiences throughout the design process. These reflective journals not only offered students an opportunity to articulate their learning experiences but also provided researchers with qualitative data.

The qualitative analysis was focused on identifying the different phases of the modeling process undertaken by the participants. This involved a detailed examination of the modeling and redesign cycles generated by the students. The analysis drew on data from the project dossiers and observational notes taken during lab sessions. These notes provided context and additional insights into the participants' interactions, problem-solving strategies, and use of technology.

3. Results

The project "*Foods of the World*" was done during year 2023/24, Figure 3 shows some of the dishes chosen by the students, such as a wine set from Valladolid (Spain), "*humita"* from Chile and a "*espeto"* (skewer) of sardines from Málaga (Spain).

Figure 3. Final productions of the 3D modeling project.

In addition to choosing a typical dish from their region, the students added different elements to complement and decorate their final presentation. For example, the student who chose wine products made a bottle of wine, a wine glass, a corkscrew, and a bunch of grapes.

3.1 An Example of a Modeling Cycle Based on a Tangible Object: The Wine Bottle

This modeling process began with the selection of the real and tangible object and its measurement, on which they would base their model. Having direct physical access to the object allowed the student to take accurate measurements, analyze proportions, and observe geometric features in detail. They made some very restrictive initial simplifications since they were not very familiar with the software either. As the modeling cycles were carried out, more complex details were added in order to obtain objects that were closer to the initial object.

Figure 4. Modeling process of the bottle.

For example, Figure 4 shows the process developed to make the wine bottle. At first, very simple figures were chosen, only cylinders and cones. Cylinders were used to make the body and the two parts of the neck of the bottle. A truncated cone was used to resemble the curved top of the body. At this stage, the inclusion of letters as a label and other details was not included.

The 3D printing of the object allows the validation of the model obtained. At this point, its resemblance to the real object is verified, and changes are proposed to be made (by the teacher or by the students themselves) either to make it more similar to the real object or to correct errors. For instance, Figure 4 shows that in the case of the bottle, the first model made using a truncated cone. In the next iteration of the cycle, it was replaced by a semi-sphere, which generated a smoother edge and a closer resemblance to the real object. Finally, after some feedback and one more modeling cycle, the letters were added as a label to give the final model. The label was made by intersecting the text with a cylinder wider than the body of the bottle to obtain the desired curvature.

Figure 5 below illustrates a simplified scheme of the modeling cycle used by the student during this process, following the Greefrath modeling cycle (Figure 2).

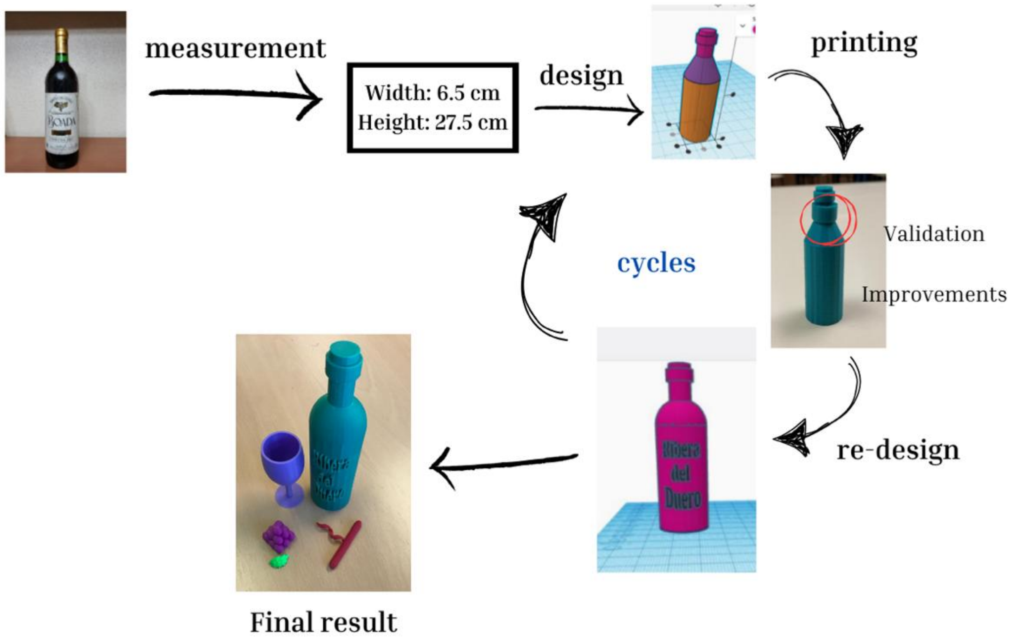

Figure 5. Modeling the cycle of the wine bottle design.

The student started by taking measurements of a real object. Using the 3D modeling tool Tinkercad, he made a preliminary design that allowed him to re-enter the modeling cycle with an improved perspective. This design was subsequently materialised through 3D printing, which facilitated the evaluation and validation of the results initially obtained. Based on this evaluation, a new modeling cycle was conducted, which allowed the wine bottle model to be further refined and detailed. The results obtained are in line with the cycle proposed by Greefrath, where 3D printing not only serves as a bridge between the mathematical results and their real-world application but also as a means of continuous iteration within the mathematical modeling process.

3.2 An Example of a Modeling Cycle Based on a Visual Reference: the Grapevine Leaf

The modeling process started with the identification of a grapevine leaf as a visual referent from a real-world context. The student initially described the object as a "leaf with serrated edges and numerous veins" and sketched a simplified drawing, noting it would be "something similar but more elongated." In this case, the absence of a tangible manipulative required careful interpretation of the visual reference, using the sketch and description as guides for the modeling process.

In the first modeling cycle using Tinkercad, the student created an oval-based cylinder representing the main body of the leaf. To simulate the serrated edges, the student intersected this main shape with several hollow cylinders. However, upon visually reviewing the resulting design, aligned with the visualization step described in Greefrath's modeling cycle, it became evident that the model lacked the intended irregularity typical of a natural grapevine leaf, prompting the need for a second modeling cycle. The student's annotations, initial sketch, and the first model construction can be found in Figure 6.

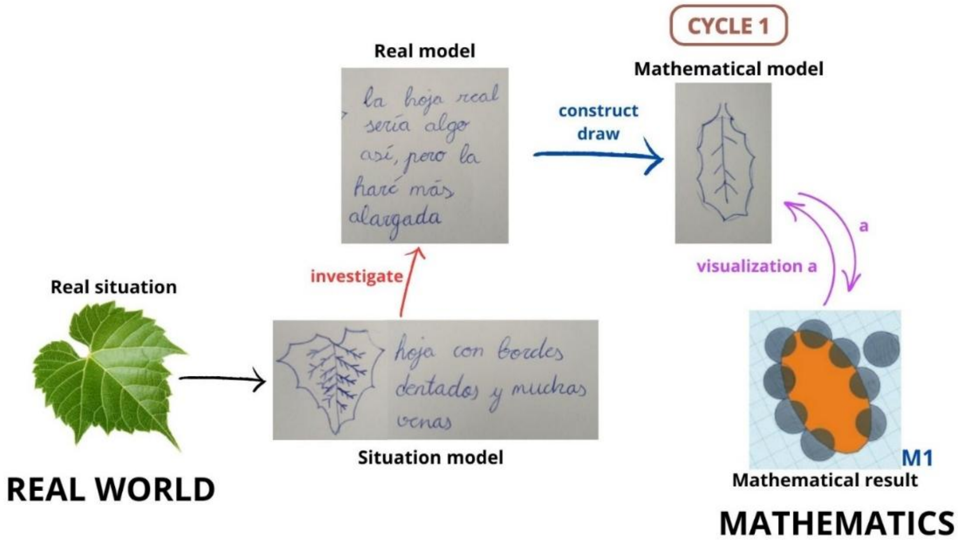

Figure 6. First modeling iteration: initial sketch, design proposal, and construction of the leaf shape.

During the second modeling iteration, the student adjusted the dimensions of the hollow cylinders to introduce greater irregularity along the leaf edges (M2). Additionally, to increase the precision and realism of the model, thin rectangular prisms were added to simulate veins, carefully rotated and resized to resemble the intricate vein structure found in grapevine leaves (M3). Once this digital model was finalized, it was 3D printed for validation. Nevertheless, the printed model (M4) revealed significant limitations: the veins were too thin to be printed clearly due to technological constraints, and the leaf itself was flat. The details of this second iteration are illustrated in Figure 7.

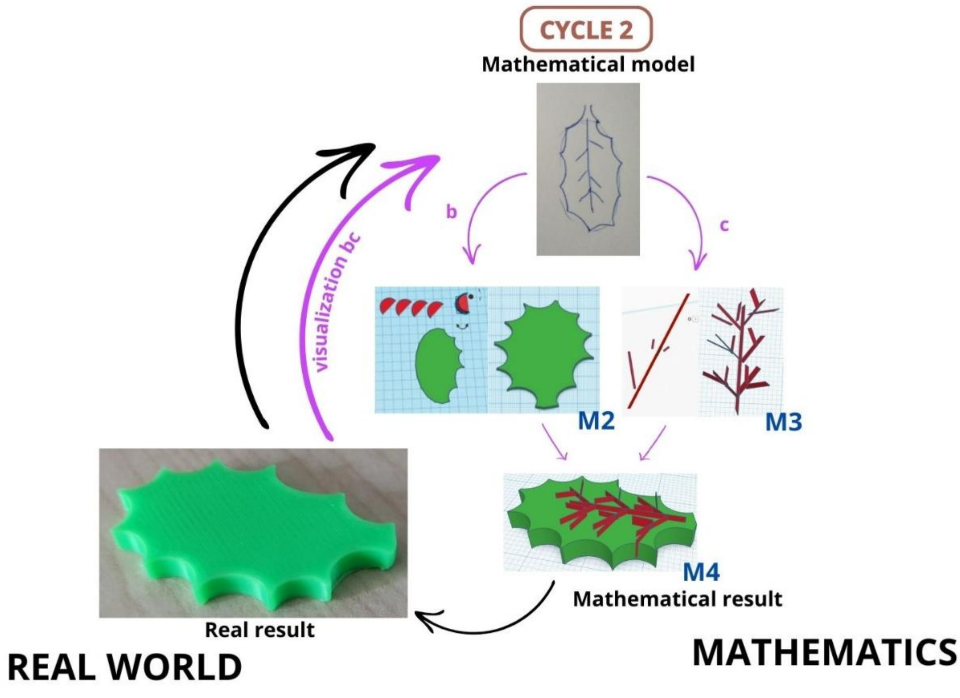

Figure 7. Second modeling iteration: refining leaf irregularities and vein structures.

These issues led to the initiation of a third and final modeling cycle. Two key adjustments were undertaken: **creating a curved** leaf (M5) and redesigning the veins (M7). To achieve curvature, the student created a mold consisting of an oval semicylinder combined with a rectangular prism fitted with a matching semicylindrical arc. The thickness of the leaf was increased to intersect it effectively with this hollow mold. Simultaneously, a new design for the veins was developed (M6), utilizing thicker components to overcome previous printing limitations. Small parallelepipeds were combined into branching structures, duplicated, rotated, and symmetrically arranged around a central parallelepiped acting as the main vein. This complete vein structure was then curved using the same mold approach employed for the leaf's curvature. The final steps of this modeling cycle are depicted in Figure 8.

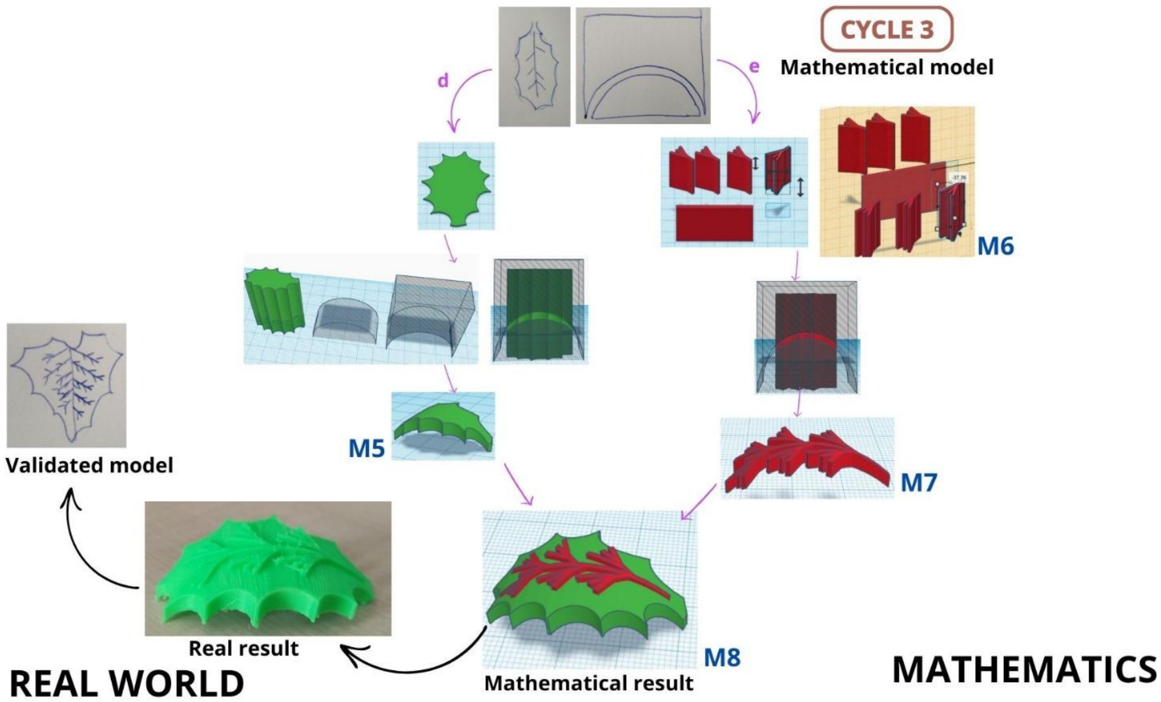

Figure 8. Third modeling cycle: introducing curvature and redesigning veins for structural accuracy and printability.

This final model (M8), after being successfully printed in 3D, was validated as accurately representing the intended grapevine leaf, effectively transitioning from a visual reference into a tangible mathematical model.

3.3 Modeling Cycles of the Whole Class

During the realization of the 3D modeling and printing project, student participation was high and, in general, students were involved. Table 1 shows the topic each student worked on, with the number of objects they designed, and the average number of modeling cycles performed. It can be observed that all the students made at least 2 modeling cycles, indeed the average of the whole class was 2.4.

Table 1. Summary of the objects and cycles realized by the students.

Project	Nos. Objects	Cycles (average)
"*Humita*" from Chile	6	2.5
"*Almogrote*" from Canary Islands	3	2.7
Calamari "*bocadillo*" from Madrid	3	3
"*Empanada*" and "*emboque*" from Chile	3	2
Chinese desserts	4	2.25
Madrilean "*cocido*"	6	2
Wine products from Valladolid	5	2
Chocolate and churros from Madrid	4	2.5
Mexican taco	4	2.25
"*Pintxo*" from the Basque Country	2	2
Canary Islands drinks	2	3
Chinese fruit brochette	6	3
"*Espeto*" (skewer) of sardines from Málaga	6	2.2

Decimal numbers appear in the number of cycles because, as the students didn´t make just one object, they did a different number of cycles for each object. So, the number shown in the table is the average of the cycles they use for all the objects.

3.4 Some Special Cases of the Modeling Cycle

Although this group of students was facing for the first time the use of Tinkercad and a 3D design process for the first time, it is worth highlighting the mathematical communication developed by the students and the spatial reasoning skills they put into operation. For example, the student who made the "*humita*" from Chile used a varied selection of three-dimensional objects and constructive geometry operations (alignment, union, difference, intersection) to make her final model: the cord surrounding the "humita" was made by a rectangular prism, which she intersected with a smaller one to obtain a frame. To curve the corners, she first used a shape called "tube" from Tinkercad, but since it was hollow in the center, the expected result was not obtained. Finally, she decided to use a torus, which allowed her to cut the desired shape (see Figure 9).

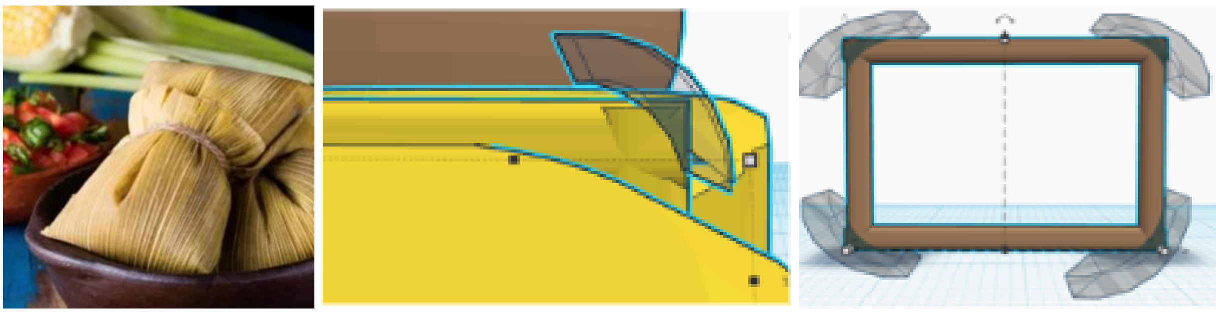

Figure 9. Rope thread bending process.

In order to obtain the reliefs that resembled the "*humita*" leaf, she tried to use joined planes, but the result obtained was not the desired one because the cuts could hardly be seen when printed. So, finally, a predefined model of a wood plank in Tinkercad had its own reliefs that allowed the intersection of both pieces to obtain the desired result (see Figure 10).

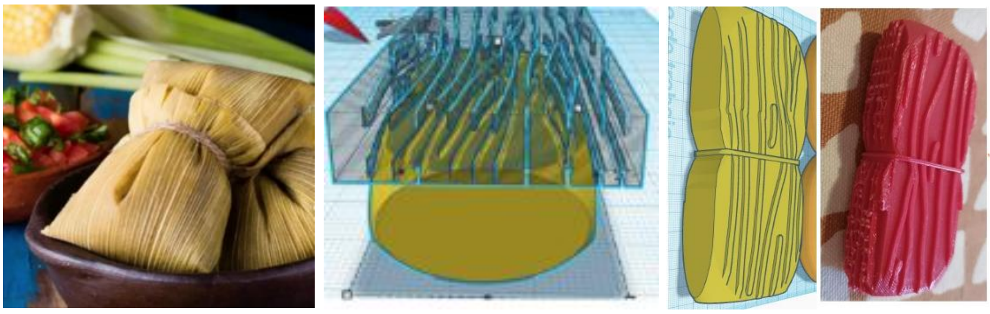

Figure 10. Image model of the real humita and relief process performed to give texture to the leaf.

It should be noted that some of the students were not satisfied with the 3D models as they came out of the printer, and they wanted to give it a more realistic touch by painting the pieces. In the case of the student who made the "*espeto*" of sardines, after painting it, she obtained a result much closer to the reality (Figure 11).

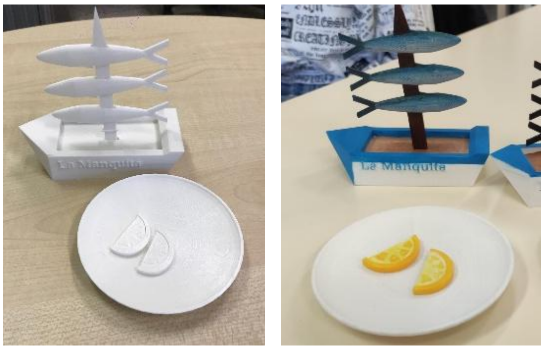

Figure 11. Final result of the "*espeto*" of sardines.

It should be noted that, in a different activity, they took advantage of the knowledge they acquired with 3D modeling and printing. In order to give Primary School Students coming to the University some gifts, they decided to create pirate medals with Tinkercad. After printing them, they also decided to paint them to have a more professional touch.

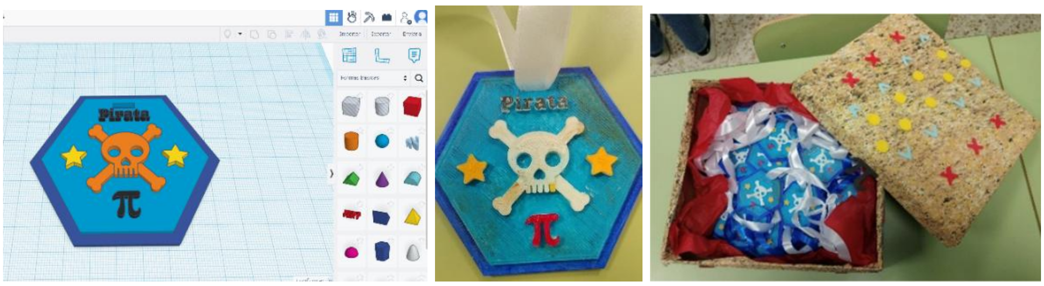

Figure 12. The pirate medals.

3.5 Students' Feedback and Perceptions of the Activity

Following the implementation of the activity, informal interviews were conducted with the students to gather their feedback. For all participants, this was their first experience with mathematical modeling, Tinkercad, and 3D printing. During the sessions, two different paces of progress emerged among the students, which led to the development of three distinct perspectives.

Most students responded enthusiastically to the experience and adapted quickly to both the software and its functionalities. Their feedback was overwhelmingly positive, and they expressed that such activities could be valuable in their future teaching practices.

A second group, comprising approximately 3 to 4 students out of 13, initially encountered difficulties in understanding and using the software. However, after a brief period of training and adaptation, they demonstrated significant progress and became highly engaged. Their feedback remained positive, though they critically highlighted the initial learning difficulty. Nevertheless, they considered the tool feasible for classroom use, provided that adequate time is allocated for familiarization.

A small minority—two students—reported a negative experience. They were critical of the tool and said that they were not going to use it in their classrooms. Despite this, they recognized its educational potential under certain conditions.

4. Conclusions

The 3D modeling and printing activity generated a high level of engagement and interest among the master's students. They were able to enhance their modeling skills through iterative cycles aimed at refining their outputs. As mentioned in [6] and [14], mathematical modeling linked abstract mathematical concepts directly to real-world applications, enhancing both understanding and practical engagement. Despite their initial limited expertise in both mathematical concepts and technological tools, the students showed significant growth and produced an outstanding mathematical work. This aligns with findings by Greefrath et al. [10] and Blum and Leiß [4], who emphasize the enhancement of cognitive processes through digital tools in the modeling cycle, making geometrical abstract concepts more tangible and comprehensible. The good results obtained from the experience support the relevance and suitability of including this type of mathematical modeling activity in teacher training. For instance, teacher education programs could incorporate paired sessions where students first model a real object based on direct measurements and later replicate a similar task using only a visual or conceptual referent. This would reflect on the different cognitive and spatial demands involved in each case.

There is evidence of the fundamental role played by visualization processes and skills during the creation of 3D models, reflecting the educational potential of these technologies to enhance participation, motivation, and the learning process, particularly within STEAM education frameworks [1,12]. Moreover, the use of CAD software, as noted in [7] and [13], supports the development of crucial spatial reasoning and visualization skills, further validating the integration of such digital tools in mathematical education. The inclusion of more guided activities and reflection on what kind of modeling projects—structured versus open-ended—and the type of referents involved—physical or visual—can foster deeper mathematical work, as well as influence students' engagement, motivation, and perception of themselves as learners, emerge as lines to be considered in future research.

5. Improvements and Future Directions

The project is still being carried out with students from the same master's program, but now with a more narrowly defined theme. This refinement aims to reduce the variability in students' final productions and to allow a more consistent analysis. In addition to qualitative data gathered through open-ended surveys and informal interviews, more structured opinion questionnaires will be administered to collect quantifiable insights into students' experiences and perceptions. One of the main challenges in the earlier stages of the project was the assessment process, due to the wide range of student outcomes. With a more focused approach, we aim to enhance both the evaluation process and the comparability of results, while still capturing individual perspectives.

Adaptations of the project have also been developed for Primary and Secondary Education and are currently being implemented in some schools. In this context, students will complete a Spatial Reasoning Instrument [15] before and after the activity, allowing quantitative analysis of potential improvements. At the same time, qualitative data will be collected through student reflections and interviews to better understand how learners engage with the activity and perceive its impact. This mixed-methods approach seeks to provide a more comprehensive understanding of the educational value and potential of integrating modeling and 3D design tools in mathematics education.

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